3. *Central Tendency*

Central Tendency and its Measures

The Central Tendency of a variable means a typical value around which other values which can be measured tend to concentrate. It is the central value or a representative value of a statistical series.

The methods of statistical analysis by which averages of the statistical series are worked out are known as Measures of Central Tendency.

> ***According to Yule and Kendall*** *"Measure of the location or position of a frequency distribution is called average".*

> ***According to Croxton and Cowden*** *"An average is a single value within the range of the data which is used to represent all the value in the series. Since an average is somewhere within the range of the data. It is sometimes called a measure of central value.*

So central objective of statistical analysis is to get one single value that describe the characteristics of the entire data.

The main objectives of Measure of Central Tendency are:

1. To condense data in a single value.
2. To facilitate comparisons between data.

Requisites of a Good Measure of Central Tendency

1. *It should be rigidly defined.*
2. *It should be simple to understand & easy to calculate.*
3. *It should be based upon all values of given data.*
4. *It should be capable of further mathematical treatment.*
5. *It should have sampling stability.*
6. *It should not be unduly affected by extreme values.*

Measures of Central Tendency

The following are the five measures of average or central tendency that are in common use :

(I) Arithmetic Mean

(II) Median

(III) Mode

(IV) Geometric Mean

(V) Harmonic Mean

Did You Know ?

Arithmetic mean, Geometric mean and Harmonic means are usually called Mathematical averages while Mode and Median are called Positional averages.

The arithmetic mean can further be divided in following two types:

(i) Simple Arithmetic Mean: In this all the values of a series are given equal importance.

(ii) Weighted Arithmetic Mean: In this different values of a series are assigned different weights in accordance with their relative importance.

(I) Arithmetic Mean

(i) Simple Arithmetic Mean : *The simple arithmetic mean is equal to the sum of all the values in the data set divided by the number of values.* So, if we have 'n' values in a data set and they have values x_1, x_2, ..., x_n, the sample mean, usually denoted by (pronounced x bar), is:

Focus Formula

$$\bar{X} = \frac{Sum\ of\ all\ Values}{Number\ of\ Values}$$

Or $$\bar{X} = \frac{\Sigma X}{N}$$

Methods of Calculating Simple Arithmetic Mean

(a) Individual Series : For individual series mean can be calculated by two methods :

Direct Method : *In this method mean is simply calculated by adding all the values of a series divided by number of values.* This method is used when value of observations is small and also number of observations are less.

Focus Formula

$$\bar{X} = \frac{Sum\ of\ all\ Values}{Number\ of\ Values}$$

Or $$\bar{X} = \frac{\Sigma X}{N}$$

Example : The pocket allowance of 10 students is Rs. 20, 19, 18, 17, 16, 15, 14, 13, 12 and 11. Find out the average pocket allowance.

Solution :

$$\bar{X} = \frac{\text{Sum of all Values}}{\text{Number of Values}}$$

$$\bar{X} = \frac{20+19+18+17+16+15+14+13+12+11}{10}$$

$$\bar{X} = \frac{155}{10}$$

$$\bar{X} = \text{Rs. } 15.5$$

Short-Cut Method : *This method is used when value of observations is large. It involves following steps :*

- *A value (in most cases from the series) is taken as 'Assumed Mean' (A). It should preferably be the middle value between maximum and minimum value of the series.*
- *Deviation of all values from this assumed mean is taken.*
  d (deviation) = X – A
- *Sum of all deviations from assumed mean is calculated.*
- *The sum of deviation from assumed mean is then divided by the number of values in the series. The following formula is then applied to calculate the required mean of the given series :*

$$\boxed{\bar{X} = A + \frac{\Sigma d}{N}}$$

Example : Following is the pocket allowance of 10 students. Find out arithmetic mean Short-cut Method.

Pocket Allowance (₹)	15	20	30	22	25	18	40	50	55	65

Solution :

(Assumed Average, A = 40)

Number of Students	Pocket Allowance (₹) (X)	Deviation from the Assumed Average (d = X-A), (A = 40)	
1	15	15 - 40 = - 25	
2	20	20 - 40 = - 20	
3	30	30 - 40 = -10	
4	22	22 - 40 = -18	-110
5	25	25 - 40 = -15	
6	18	18 - 40 = -22	
7	40(A)	40 - 40 = 0	
8	50	50 - 40 = +10	
9	55	55 - 40 = + 15	+50
10	65	65 - 40 = + 25	
N = 10		∑d = -110+50 = -60	

The Sum of '+' deviations = + 50

The sum of '–' deviations = – 110

The net sum of deviation, $\Sigma d = -110 + 50 = -60$

Dividing the aggregate of deviations (Σd) by the number of items (N),

$$\frac{\Sigma d}{N} = \frac{-60}{10} = -6$$

Substituting this value of $\frac{\Sigma d}{N}$ in the following formula:

$$\bar{X} = A + \frac{\Sigma d}{N}$$

We have,

$$\bar{X} = 40 + (-)\ 6$$
$$= 40 - 6 = 34$$

Arithmetic Mean = ₹ 34.

(b) Discrete Frequency Series : In discrete frequency series, there are frequencies corresponding to different items in the series. The mean can be calculated from any of the following three methods :

- Direct Method
- Short-cut Method
- Step-deviation Method

Direct Method : *This method involves the following step :*

- *Each value of X is multiplied by the corresponding value of the frequency f to get ΣfX.*
- *The values of fX are added to get .*
- *The values of frequencies are added to get Σf. This value is the total number of observations in the series.*
- *The value of ΣfX is divided by Σf to get the required mean.*

$$\boxed{\bar{X} = \frac{\Sigma fx}{\Sigma f}}$$

Example : Following is the weekly wage earning of 19 workers :

Wages (₹)	10	20	30	40	50
Numbers of Workers	4	5	3	2	5

Calculate arithmetic mean using Direct Method.

Solution :

Wages ₹ (X)	Number of Workers or Frequency (f)	Multiple of the Value of X and Frequency (fx)
10	4	4 x 10 = 40
20	5	5 x 20 = 100
30	3	3 x 30 = 90
40	2	2 x 40 = 80
50	5	5 x 50 = 250
	$\sum f = 19$	$\sum fX = 560$

$$\overline{X} = \frac{\Sigma fX}{\Sigma f} = \frac{560}{19} = 29.47$$

Mean wage earnings of 19 workers = ₹ 29.47

Short-cut Method : *This method involves the following step :*

- *A value (in most cases from the series) is taken as 'Assumed Mean' (A). It should preferably be the middle value between maximum and minimum value of the series.*
- *Deviation of all values from this assumed mean is taken.*

$$d\,(\text{deviation}) = X - A$$

- *The value of d (deviation) is multiplied by the corresponding value of frequency f to get fd. All the values of fd are then added to get Σfd.*
- *The values of frequencies are added to get Σf. This value is the total number of observations in the series.*
- *The value of Σfd is divided by Σf. The mean is then obtained by the following formula.*

$$\boxed{\overline{X} = A + \frac{\Sigma fd}{\Sigma f}}$$

Example : Following are the wages of 19 workers :

Wages ₹	10	20	30	40	50
Numbers of Workers	4	5	3	2	5

Calculate arithmetic mean, using 'Short-cut Method'.

Solution :

(Assumed Average, A = 30)

Wages ₹ (X)	**Number of Workers or Frequency (f)**	**Deviation (d = X – A) (A = 30)**	**Multiple of deviation and Frequency (fd)**
10	4	10 – 30 = – 20	4 x (– 20) = – 80 } - 130
20	5	20 – 30 = – 10	5 x (– 10) = – 50
30	3	30 – 30 = 0	3 x 0 = 0
40	2	40 – 30 = 10	2 x 10 = 20 } + 120
50	5	50 – 30 = 20	5 x 20 = 100
	∑f = 19		∑fd = – 130 + 120 = – 10

$$\overline{X} = A + \frac{\Sigma fd}{\Sigma f} = 30 + \frac{-10}{19} = 30 - \frac{10}{19}$$

$= 30 - 0.53 = 29.47$

Arithmetic Mean = ₹ 29.47

Step-deviation Method : *This method is an extension of short-cut method. It can be adopted when deviation from the assumed mean have some common factor. This common factor is indicated by 'C'. The deviation (d) when divided by the common factor C is called step-deviation. This is indicated by d. This method involves the following step:*

- *A value (in most cases from the series) is taken as 'Assumed Mean' (A). It should preferably be the middle value between maximum and minimum value of the series.*
- *Deviation of all values from this assumed mean is taken.*
  $d\,(deviation) = X - A$
- *Step deviation d' is obtained by dividing the deviation (d) by the common factor.*

$$d' = \frac{X - A}{C} = \frac{d}{C}$$

- *Each step deviation is multiplied with the corresponding frequency to get fd'. The values of fd' are then added to get Σfd'.*
- *The values of frequencies are added to get Σf. This value is the total number of observations in the series.*
- *The values of Σfd' is then divided by the corresponding values of frequency f. The mean is then obtained by the following formula.*

$$\boxed{\overline{X} = A + \frac{\Sigma fd'}{\Sigma f} \times C}$$

Example : Wage rate of 19 workers is given below :

Wages ₹	10	20	30	40	50
Numbers of Workers	4	5	3	2	5

Calculate arithmetic mean using 'Step-deviation Method.

Solution :

(Assumed Average, A = 30)

Wages ₹ (X)	**Number of Workers or Frequency (f)**	**Deviation (d = X – A) (A = 30)**	**Step-Deviation $\left(d' = \frac{X - A}{C}\right)$ (C = 10)**	**Multiple of Step-Deviation and Frequency (fd')**
10	4	10 – 30 = – 20	$\frac{-20}{10} = -2$	4 × (–2) = – 8
20	5	20 – 30 = – 10	$\frac{-10}{10} = -1$	5 × (–1) = – 5
30	3	30 – 30 = 0	$\frac{0}{10} = 0$	3 × 0 = 0
40	2	40 – 30 = 10	$\frac{10}{10} = 1$	2 × 1 = 2
50	5	50 – 30 = 20	$\frac{20}{10} = 2$	5 × 2 = 10
	∑f = 19			∑fd' = -1

$$\overline{X} = A + \frac{\Sigma fd'}{\Sigma f} \times C$$

A = 30, C = 10

and $\frac{\Sigma fd'}{\Sigma f} = \frac{-1}{19} = -0.053$

Putting these values in the formula:

$$\overline{X} = 30 + \frac{-1}{19} \times 10 = 30 - 0.053 \times 10$$

$$= 30 - 0.53 = 29.47$$

Arithmetic Mean = ₹ 29.47

(c) Continuous Frequency Distribution

In case of continuous frequency distribution values are classified into various class intervals like 0-10, 10-20 etc. The first number of each class is called the lower class limit and the second number is called the upper class limit. For example, in the class 0-10, 0 is the lower class limit and 10 is the upper class limit. The mean can be calculated from any of the following three methods:

1. *Direct Method*
2. *Short-cut Method*
3. *Step-deviation Method*

Direct Method : *This method involves the following steps :*

- *The mid value (m) of each class is calculated by adding the lower and upper class limit and then dividing the sum by 2. For example, the mid value of the class 0-10 is*

  $$m = \frac{0+10}{2}$$

  $$\text{or} \qquad m = \frac{I_1 + I_2}{2}$$

  Here, m = *Mid-value of the class*

  I_1 = *Lower limit of the class*

  I_2 = *Upper limit of the class*

- *Mid-values are multiplied by their corresponding class frequencies to get fm.*
- *The values of fm are added to get* Σfm.
- *The values of frequencies are added to get* Σf. *This value is the total number of observations in the series.*
- Σfm *is divided by* Σf *to get the required mean.*

$$\boxed{\bar{X} = \frac{\Sigma fm}{\Sigma f}}$$

Example : The following table shows marks in English secured by students of Class X in your school in their examination. Calculate mean arks using Direct Method.

Marks	0-10	10-20	20-30	30-40	40-50
Numbers of Students	20	24	40	36	20

Solution :

Marks	Mid-value $\left(m = \frac{l_1 + l_2}{2}\right)$	Number of Students of Frequency (f)	Multiple of Mid-value and Frequency (fm)
0-10	$\frac{0+10}{2} = 5$	20	20 × 5 = 100
10-20	$\frac{10+20}{2} = 15$	24	24 × 15 = 360
20-30	$\frac{20+30}{2} = 25$	40	40 × 25 = 1,000
30-40	$\frac{30+40}{2} = 35$	36	36 × 35 = 1,260
40-50	$\frac{40+50}{2} = 45$	20	20 × 45 = 900
		$\sum f = 140$	$\sum fm = 3,620$

$$\overline{X} = \frac{\Sigma fm}{\Sigma f} = \frac{3,620}{140} = 25.86$$

Mean Marks = 25.86

Short-cut Method : *This method involves the following steps :*

- *Mid-value (m) of each class is calculated as discussed in the direct method.*
- *A value is selected as assumed mean (A) which is preferable be the middle value between maximum and minimum value of mid-values.*
- *Deviations (d) of all mid-values are taken from the assumed mean.*
- *The deviations (d) values are multiplied by the corresponding values of frequency to get fd.*
- *All values of fd are added to get Σfd.*
- *The values of frequencies are added to get Σf. This value is the total number of observations in the series.*
- *Σfd is divided by Σf. The following formula is then used to calculate the required mean.*

$$\boxed{\overline{X} = A + \frac{\Sigma fd}{\Sigma f}}$$

Example : The Following table shows marks secured by the students of a class in an examination in English:

Marks	0-10	10-20	20-30	30-40	40-50
Numbers of Students	20	24	40	36	20

Calculate mean marks using Short-cut Method.

Solution :

(Assumed Average, A = 25)

Marks	Mid-value $\left(m = \frac{l_1 + l_2}{2}\right)$	Number of Students of Frequency (f)	Deviation (d = m – A) (A = 25)	Multiple of Deviation and Frequency (fd)
0-10	$\frac{0+10}{2} = 5$	20	5 – 25 = -20	20 x –20 = – 400
10-20	$\frac{10+20}{2} = 15$	24	15 – 25 = -10	24 x –10 = – 240
20-30	$\frac{20+30}{2} = 25$	40	25 – 25 = 0	40 x 0 = 0
30-40	$\frac{30+40}{2} = 35$	36	35 – 25 = +10	36 x 10 = + 360
40-50	$\frac{40+50}{2} = 45$	20	45 – 25 = +20	20 x + 20 = + 400
		Σf = 140		Σfd = 120

$$\overline{X} = A + \frac{\Sigma fd}{\Sigma f} = 25 + 0.86 = 25.86$$

Mean Marks = 25.86

Step-Deviation Method : *This method is an extension of short-cut method. It can be adopted when deviation from the assumed mean have some common factor. This common factor is indicated by 'C'. The deviation (d) when divided by the common factor C is called step-deviation. This is indicated by d'. This method involves the following step :*

- *Mid-value (m) of each class is calculated as discussed in the direct method.*
- *A value is selected as assumed mean (A) which is preferable be the middle value between maximum and minimum value of the calculated mid-values.*
- *Deviation of all mid-values from this assumed mean is taken.* $d(\text{deviation}) = m - A$
- *Step deviation d' is obtained by dividing the deviation (d) by the common factor.*

$$d' = \frac{m-A}{C} = \frac{d}{C}$$

- *Each step deviation is multiplied with the corresponding frequency to get fd'. The values of fd' are then added to get Σfd'.*
- *The values of frequencies are added to get Σf. This value is the total number of observations in the series.*
- *The values of Σfd' is then divided by the corresponding values of frequency f. The mean is then obtained by the following formula.*

$$\boxed{\bar{X} = A + \frac{\Sigma fd'}{\Sigma f} x C}$$

Example : The Following table shows marks obtained by the students of a class in their test in English:

Marks	0-10	10-20	20-30	30-40	40-50
Numbers of Students	20	24	40	36	20

Calculate arithmetic mean using Step-deviation Method.

Solution : **(Assume Average, A = 25)**

Marks	Mid-value $\left(m = \frac{l_1 + l_2}{2}\right)$	Number of Students or Frequency (f)	Deviation (d = m – A) (A = 25)	Step-deviation (d') $\left(d' = \frac{m - a}{C}\right)$ (C = 10)	Multiple of Step- deviation and Frequency (fd')
0-10	5	20	–20	–2	–40 $\}$ - 64
10-20	15	24	–10	–1	–24
20-30	25	40	0	0	0
30-40	35	36	+10	1	+36 $\}$ 76
40-50	45	20	+20	2	+40
		$\sum f = 140$			$\sum fd' = 12$

$$\frac{\Sigma fd'}{\Sigma f} = \frac{12}{140} = 0.086$$

$$\bar{X} = A + \frac{\Sigma fd'}{\Sigma f} \times C$$

= 25 + 0.086 × 10 = 25 + 0.86

Mean Marks = 25.86

Calculation of Corrected Arithmetic Mean

Sometimes the values are wrongly written in the calculation of Arithmetic Mean. Accordingly, the mean value goes wrong. In such case the corrected mean value can be calculated as follows:

$$\boxed{\bar{X} = \frac{\sum X(\text{wrong}) + (\text{Correct values}) - (\text{Incorrect values})}{N}}$$

Example : Mean marks obtained by 100 students are estimated to be 40. Later on it is found that one value was read as 83 instead of 53.

Solution :

$$\overline{X} = \frac{\Sigma X}{N}$$

or, $\Sigma X = \overline{X}N$

Given : $\overline{X} = 40$; N = 100

$$40 = \frac{\Sigma X}{100}$$

$\therefore$ $\Sigma X(\text{wrong}) = 40 \times 100 = 4{,}000$

Correct value = 53

Incorrect value = 83

Correct

$$\overline{X} = \frac{\Sigma X(\text{wrong}) + (\text{Correct value}) - (\text{Incorrect value})}{N}$$

$$= \frac{4{,}000 + 53 - 83}{100}$$

$$= \frac{3970}{100} = 39.70$$

Thus, Correct Mean = 39.70

Combined Arithmetic Mean

If two groups are combined, then the Arithmetic Mean of the whole group can be calculated using the formula:

Focus Formula

$$\overline{X}_{1,2} = \frac{\overline{X}_1 N_1 + \overline{X}_2 N_2}{N_1 + N_2}$$

Here, $\overline{X}_{1,2}$ = *Combined arithmetic mean of parts 1 and 2 of a series.*

$\overline{X}_1$ = *Arithmetic mean of part 1 of the series.*

$\overline{X}_2$ = *Arithmetic mean of part 2 of the series.*

N_1 = *Number of items in part 1 of the series and*

N_2 = *Number of items in part 2 of the series.*

Likewise, when there are more then 2 parts of a series, the following formula is used to work out Combined Arithmetic Mean.

$$\boxed{\overline{X}_{1,2,3,\ldots,n} = \frac{\overline{X}_1 N_1 + \overline{X}_2 N_2 + \ldots + \overline{X}_n N_n}{N_1 + N_2 + \ldots + N_n}}$$

Example : 60 students of Section A of Class XI, obtained 40 mean marks in Statistics, 40 students of Section B obtained 35 mean marks in Statistics. Find out mean marks in Statistics for Class XI as a whole.

Solution : Given : $N_1 = 60$, $\overline{X}_1 = 40$, $N_2 = 40$, $\overline{X}_2 = 35$

We know,

$$\overline{X}_{12} = \frac{\overline{X}_1 N_1 + \overline{X}_2 N_2}{N_1 + N_2}$$

$$\therefore \quad \overline{X}_{12} = \frac{40 \times 60 + 35 \times 40}{60 + 40}$$

$$\therefore \quad \overline{X}_{12} = \frac{2,400 + 1,400}{100} = \frac{3,800}{100} = 38$$

Thus, Combined Arithmetic Mean = 38.

Some Properties of Mean

(i) The sum of the deviations of all the values of a series from their Arithmetic Mean is always zero, i.e.

$$\Sigma(X - \overline{X}) = 0$$

(ii) If a number is added, subtracted, multiplied or divided to all the values of a series then their mean will also get changed by the same value.

If a constant amount is added, subtracted, multiplied or divided, the arithmetic mean will be affected accordingly. Check the following illustration:

X	X + 2	X – 2	X × 2	$\frac{X}{2}$
10	12	8	20	5
12	14	10	24	6
14	16	12	28	7
16	18	14	32	8
$\Sigma x = 52$	$\Sigma(x+2) = 60$	$\Sigma(x-2) = 44$	$\Sigma(x+2) = 104$	$\Sigma(x-2) = 26$
$\overline{x} = 13$	$\overline{x} = 15$	$\overline{x} = 11$	$\overline{x} = 26$	$\overline{x} = 6.5$

(ii) Weighted Arithmetic Mean : In simple arithmetic mean all the values of a series are given equal importance ,but sometime we have to give greater significance to some values and less to others. *When different values of a series are weighed according to their relative importance, the average of such series is called Weighted Arithmetic Mean.* It is calculated as follows:

$$\boxed{\overline{X}w = \frac{\Sigma WX}{\Sigma W}}$$

(II) Median

The median is value of the centrally located term, when the values of a given series are arranged in the ascending or descending order of magnitudes. In other words, the median is value of the variety for which total of the frequencies above this value is equal to the total of the frequencies below this value.

> ***According to Due to Corner*** *"The median is the value of the variable which divides the group into two equal parts one part comprising all values greater, and the other all values less than the median".*

Methods of Calculating Median

To locate the median, all values of a series must be arranged in either the ascending order or descending order.

(i) Discrete Series : Calculation of median in case of a discrete series involves the following steps :

(a) Arrange the values of the series in the ascending order or descending order.

(b) If the number of terms in the series are odd, median will be :

Focus Formula

$$M = \text{Size of} \left(\frac{N+1}{2}\right) \text{th item}$$

Here $M = \text{Median}$

$N = \text{Number of items.}$

If the number of terms in the series are even, median will be the value of,

$$M = \frac{\text{Size of} \left(\frac{N}{2}\right) \text{th item} + \text{Size of} \left(\frac{N}{2}+1\right) \text{th item}}{2}$$

Example : The following series show mark in Economics of 11 students of Class XI.

Marks	17	32	35	33	15	21	41	32	11	10	20

Solution :

Ascending Order		Descending Order	
S.No.	**Marks**	**S.No.**	**Marks**
1	10	1	41
2	11	2	35
3	15	3	33
4	17	4	32
5	20	5	32
6(M)	21	6(M)	21
7	32	7	20
8	32	8	17
9	33	9	15
10	35	10	11
11	41	11	10
N = 11		N = 11	

$$M = \text{Size of } \left(\frac{N+1}{2}\right)\text{th item}$$

$$= \text{Size of } \left(\frac{11+1}{2}\right)\text{th item}$$

= Size of 6th item = 21

Median = 21.

(ii) Discrete Frequency Series : Calculation of median in case of a discrete frequency series involves the following steps:

(a) Arrange the values of the series in the ascending order or descending order.

(b) Convert the simple frequency series into cumulative frequency series.

(c) Calculate the number of observations in the series which is equal to $\sum f$.

(d) If the number of terms in the series are odd, median will be,

$$M = \text{Size of } \left(\frac{N+1}{2}\right)\text{th item}$$

(Here, M = Median, N = Number of items.)

(e) If the number of terms in the series are even, median will be,

$$M = \frac{\text{Size of } \left(\frac{N}{2}\right)\text{th item} + \text{Size of } \left(\frac{N}{2}+1\right)\text{th item}}{2}$$

Example : Find the median of the following series :

Size	2	3	4	5	6	7	8	9	10
Frequency	2	3	8	10	12	16	10	8	6

Solution : Estimation of the Median

Size	Frequency	Cumulative Frequency
2	2	2
3	3	2 + 3 = 5
4	8	5 + 8 = 13
5	10	13 + 10 = 23
6	12	23 + 12 = 35
7	16	35 + 16 = 51 (M)
8	10	51 + 10 = 61
9	8	61 + 8 = 69
10	6	69 + 6 = 75
	N = 75	

Median or M

$$= \text{Size of } \left(\frac{N+1}{2}\right) \text{th item}$$

$$= \text{Size of } \left(\frac{75+1}{2}\right) \text{th item}$$

= Size of 38th item

(iii) Continuous Frequency Distribution : Calculation of median in case of a Continuous Frequency Distribution involves the following steps :

(a) Arrange the data in ascending or descending orders of their class interval.

(b) Convert the class frequencies into cumulative frequencies.

(c) Calculate the number of observations which is equal to $\sum f$.

(d) Locate the median class which correspond to that cumulative which includes the (N/2)th observation.

(e) Apply the following formula to calculate the median value :

$$\boxed{M = \ell_1 + \frac{\frac{N}{2} - c.f.}{f} \times i}$$

Here, ℓ_1 = Lower limit of the median class

c.f. = Cumulative frequency of the class preceding the median class

f = frequency of the median class

i = size of the median class interval

Example : Find out the median value of the following distribution.

Wage Rate ₹	0-10	10-20	20-30	30-40	40-50
Numbers of Workers	22	38	46	35	20

Solution :

Wage Rate ₹	**Frequency (f)**	**Cumulative Frequency**
0-10	22	22
10-20	38	60 (c.f.)
(l_1) 20-30	46(f)	106
30-40	35	141
40-50	20	161
	$\sum f = N = 161$	

$\therefore$ $M = \text{Size of } \left(\frac{N}{2}\right) \text{th item, } N = \sum f = 161$

$\therefore$ $M = \text{Size of } \left(\frac{161}{2}\right) \text{th item = Size of 80.5th item}$

80.5th item lies in 106th cumulative frequency. The class interval corresponding to this cumulative frequency is 20-30, which, therefore, is the median class interval. That is, the value of the median must lie within the range of 20-30. The following formula is applied to identify the exact value of the median.

$$M = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i$$

$$= 20 + \frac{\frac{161}{2} - 60}{46} \times 10 = 20 + \frac{80.5 - 60}{46} \times 10$$

$$= 20 + \frac{20.5}{46} \times 10 = 20 + 4.46 = 24.46$$

Median = ₹24.46

Graphic Determination of Median

Median value for a given set of data can also be determined graphically by presenting the data in the form of ogives. It can be done in two ways:

(a) By presenting the data graphically in the form of **'less than'** or **'more than'** ogives.

(b) By presenting the data graphically simultaneously in the form of **'less than'** and **'more than'** ogives on the same graph.

(a) 'Less than' and 'More than' Ogive Method

According to this approach, a frequency distribution series is first converted into a 'less than' or 'more than' cumulative series as in the case of ogives. Data are presented graphically to make a 'less than' or 'more than' ogive (N/2)th item of the series is determined and from this point (on the Y-axis of the graph) a perpendicular is drawn to the right to cut the cumulative frequency curve. The median value of the series is the one where the cumulative frequency curve is cut by the perpendicular corresponding to the X-axis.

Example : Determine median value of the following series using graphic method.

Marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Numbers of Students	4	6	10	10	25	22	18	5

Solution : Using 'less than' ogive approach, the calculation of the median value involves the following steps :

(1) Convert the series into a 'less than' cumulative frequency distribution and graph the data as under.

Estimation of the Median: 'Less than' Ogive Approach

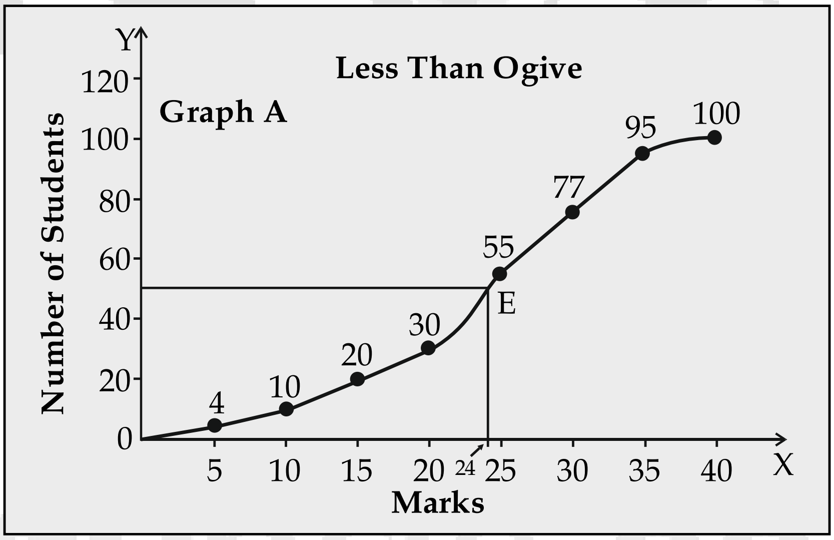
Marks	Cumulative Frequency
Less than 5	4
Less than 10	10
Less than 15	20
Less than 20	30
Less than 25	55
Less than 30	77
Less than 35	95
Less than 40	100

(Ans. Median = 24)

(2) Find out $\left(\frac{N}{2}\right)$th item and mark it on the Y-axis. In the above illustration $\left(\frac{N}{2}\right)$th item is $\frac{100}{2} = 50$.

(3) Draw a perpendicular from 50 to the right to cut the cumulative frequency curve (ogive) at point E.

(4) From the point E where cumulative frequency curve (ogive) is cut, draw a perpendicular on X-axis. The point at which it touches X-axis will be the median value of the series, as shown in Graph A :

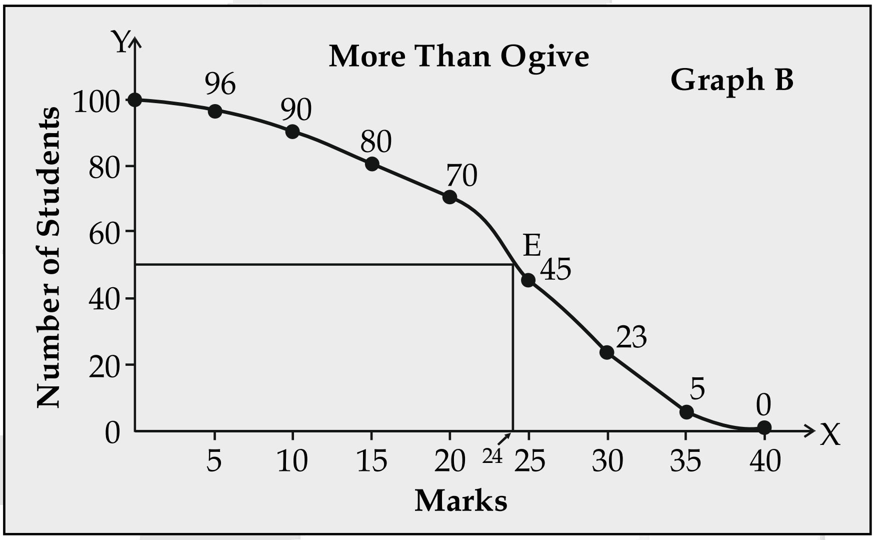


Using 'more than' ogive approach, the calculation of the median involves the similar procedure, as should be evident from the following table and Graph B :

Estimation of the Median : 'More than' Ogive Approach

Marks	Cumulative Frequency
Less than 0	100
Less than 5	96
Less than 10	90
Less than 15	80
Less than 20	70
Less than 25	45
Less than 30	23
Less than 35	5
Less than 40	0

(Ans. Median = 24)



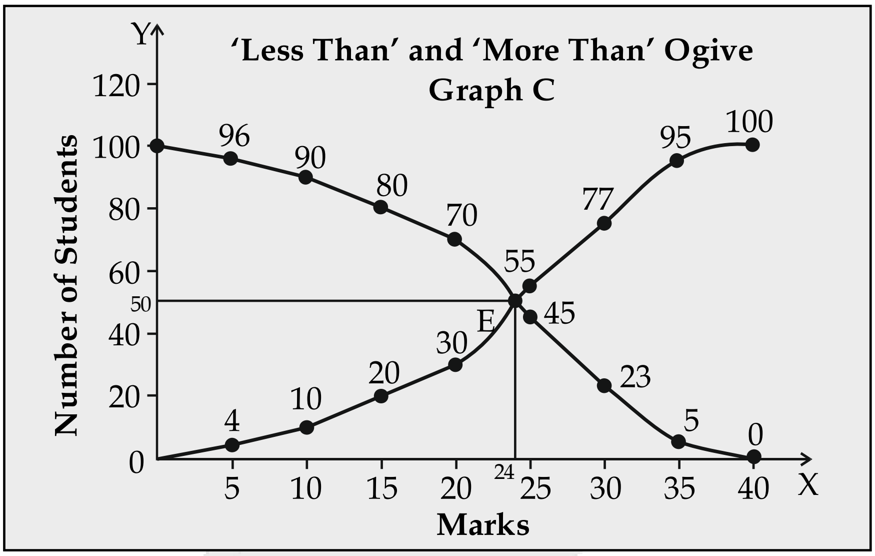
(b) 'Less than' and 'More than' Ogive Approach

Another way of the graphic determination of the median is simultaneous graphic presentation of both the 'less than' and 'more than' ogives. Mark the point E where the ogive curves cut each other, draw a perpendicular from that point on the X-axis, the corresponding value on the X-axis would be the median value.

Estimation of the Median: 'Less than' and 'More then' Ogive Approach

Marks	Cumulative Frequency	Marks	Cumulative Frequency
Less than 5	4	More than 0	100
Less than 10	10	More than 5	96
Less than 15	20	More than 10	90
Less than 20	30	More than 15	80
Less than 25	55	More than 20	70
Less than 30	77	More than 25	45
Less than 35	95	More than 30	23
Less than 40	100	More than 35	5
		More than 40	0

(Ans. Median = 24)



(III) Mode

The word **"mode"** is formed from the French word **'which means in fashion'**. It is the value which occurs most frequently in the series, i.e. the value with the highest frequency in the series is the modal value.

> **According to Dr. A. L. Bowlly** *"the value of the graded quantity in a statistical group at which the numbers registered are most numerous, is called the mode or the position of greatest density or the predominant value."*

> **According to Scholar** *"The value of the variable which occurs most frequently in the distribution is called the mode."*

The mode of a distribution is the value around the items tends to be most heavily concentrated. It may be regarded at the most typical value of the series.

Methods of Calculating Mode

(i) Discrete Series

For discrete series mode can be calculated in the following two ways:

(A) By inspection: *By inspecting the values in the series, we can find out the value which occurs most frequently.* This will be the modal value.

Example : Age of 15 students of a class is reported below. Find the modal age.

Age (Years)	22	24	17	18	17	19	18	21	20	21	20	23	22	22	22

Solution : Arrange the series in an ascending order as :

Age (Years)	17	17	18	18	19	20	20	21	21	22	22	22	22	23	24

An inspection of the series show that 22 occurs most frequently in the seres.

Hence, Mode (Z) = 22.

(B) By Converting Individual Series into Discrete Frequency Series : *When the number of observations in the series is large, mode can be calculated by converting the individual series into discrete frequency series. The value the observation with the highest frequency is then the modal value.*

Example : The table below presents death rate of population across different countries. Find the mode.

Death Rate (Per Thousand)	11.1	10.9	10.7	11.1	10.6	11.3	10.6
	10.7	10.6	10.9	10.6	10.5	10.4	10.6

Solution : First, we convert the series into a discrete frequency distribution in ascending order as under.

Death Rate	10.4	10.5	10.6	10.7	10.9	11.1	11.3
Frequency	1	1	5	2	2	2	1

10.6 is the value with highest frequency of 5. Hence,

Mode (Z) = 10.6

(ii) Continuous Frequency Distribution

In case of continuous frequency distribution, modal class is located which is the class with the highest frequency. The modal value is then calculated using the following formula

Focus Formula

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

Here Z = *Value of the mode*

l_1 = *lower limit of the modal class*

f_1 = *The frequency of the modal class*

f_0 = *The frequency of pre-modal class*

f_2 = *Frequency of the next higher class or post-modal class*

i = *Size of the modal group.*

Example : Calculate mode from the following data.

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	2	5	7	5	2

Solution :

Calculation of Mode	
Class Interval	**Frequency**
0-10	2
10-20	5 (f_0)
(l_1) 20-30	7 (f_1)
30-40	5 (f_2)
40-50	2

A glance at the series reveals that 20-30 is the modal class because it has the maximum frequency, i.e., following formula is used to calculate modal value.

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$$= 20 + \frac{7-5}{2(7)-5-5} \times 10$$

$$= 20 + \frac{2}{14-10} \times 10$$

$$= 20 + \frac{2}{4} \times 10$$

$= 20 + 5 = 25$

Mode Z = 25.

Graphic Method of Locating Mode

Mode of a series can also be located graphically. It involves the following steps :

(i) Present the given information in the form of a histogram. Identify the highest rectangle. This corresponds to model class of the series.

(ii) Join the top corners of the modal rectangle with the immediate next corners of the adjacent rectangles. The joining lines must cut each other.

(iii) The point where the joining lines cut each other points to the modal value. This value is determined by drawing a perpendicular from that point on the X-axis.

Did You Know ?

Relationship among Mean, Median and Mode

$$Mode = 3\,Median - 2\,Mean$$

$$Z = 3M - 2\overline{X}$$

Example : If in an asymmetrical distribution, median is 280 and mean is 310, what will be the mode ?

Solution :

Mode = 3 Median – 2 Mean

Mode = 3 × 280 – 2 × 310

= 840 – 620

= 220

When mode is 220 and Median is 280, Mean would be.

220 = 280 × 3 – 2 Mean

2 Mean = 840 – 220

2 Mean = 620

Mean = $\frac{620}{2}$ = 310

When Mode is 220 and Mean is 310. Median would be.

220 = 3 Median – 2 × 310

220 = 3 Median – 620

3 Median = 840

Median = $\frac{840}{3}$ = 280